4. RHEOELECTRIC ANALOGY

4.1 Rheoelectric tank for transonic flow analogy

The structure of the particular solutions used for the illustrated examples gives information also about the details of the mapping near the sonic line, $\upsilon = 0$. It is easy to verify, that a solution for ϕ, Ψ of system (8), describing an element of curved flow in transition from subsonic to supersonic flow, or reverse, is described by the locally valid expansion in ζ_0

$$\phi(\upsilon, \vartheta) = \phi^*(\vartheta) + c' \cdot \upsilon^{4/3} \cdot \dot{\Psi}^*(\vartheta) + O(\upsilon^2, \vartheta) \quad (36\ a)$$

$$\Psi(\upsilon, \vartheta) = \Psi^*(\vartheta) + c'' \cdot \upsilon^{2/3} \cdot \dot{\phi}^*(\vartheta) + O(\upsilon^2 + \vartheta) \quad (36\ b)$$

This is a very weakly singular behavior into the υ-direction, it is a consequence of generalized axisymmetric potential theory mentioned earlier and knowledge of this structure enables us to avoid certain difficulties occurring when a solution (8) or (10) in ζ has to be evaluated. The following description of transonic rheoelectric analogy mainly concentrates on outlining a technical solution for problems stemming from this singularity.

From definitions (25), (26) for the two types of analogy we see that local thickness h of a plane conductor, multiplied by its conductivity λ, is analogous to the coefficient K in analogy B, or to the reciprocal K^{-1} in analogy A. As we see directly from the near sonic equations (19), the coefficient has a cubic root zero at the sonic linewhich leads to the above mentioned exponents 4/

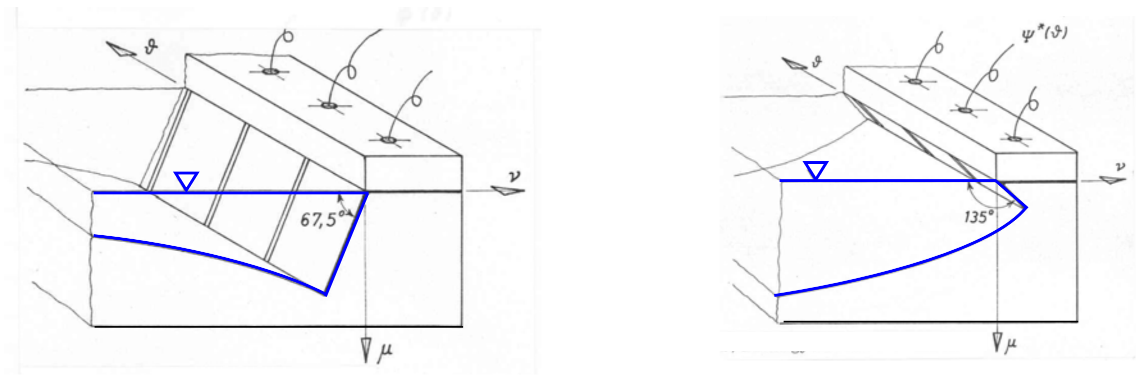
$$K(\upsilon \to 0) = K^* \sim |\upsilon|^{1/3} \quad (37)$$

3 and 2/3 occurring in expansion (36). The necessary requirement of a slowly varying thickness h of a plane conductor for validity of the plane Beltrami system (24) for the electrical variables is therefore not fulfilled near the sonic line: conductor thickness would have to go to infinity or to zero with steep gradient.

Almost classical applications of the analogy to simulate aerodynamic problems some decades ago include the compressible flow hodograph for subsonic flow[8,19], but less actuality of transonic flow at this time in general and the above mentioned limitations stemming from the zero in the coefficient (37) prohibited an efficient extension of the analogy into the Mach number unity regime.

The author's research on the aforementioned analytical structure of transonic flow in the modified hodograph plane, especially relation (37) led to a practical design of a new electrolytic tank - with water used as conductor - which allows an electric continuation of the analog flow beyond the sonic line in a Rheograph ζ. The basic idea is the use of an inclined wall boundary for the tank simulation of the sonic line, as shown in Fig. 12. The idea is the use of a locally three-dimensional electric potential to be evaluated on the surface. For Analogy A , Fig. 12a, sonic line electrodes are inclined forming a 67.5 degree wedge of the water body in the $(\upsilon, \vartheta, \mu)$-space. On the surface $\mu = 0$, where electric potential is evaluated, exponent 4/3 for representation of (36a) is observed and understood easily as a result of the local potential distribution in the (υ, μ)-plane.

In Fig. 12b the idea is illustrated for Analogy B: here we have an undercut sonic line with a 135° water body to represent the exponent 2/3 in (36 b).



a b

Figure 12. Electrolytic tank for transonic flow analogies: For velocity potential (a), for stream function (b)

The idea is, like some of the analytical properties of transonic flow in our Rheograph plane, a generalization of axisymmetric potential distribution: the "Inclined Electrolytic Tank" or a hyperbolic shape bottom with a 45 degree wedge water body is familiar to researchers having used the analogy to represent incompressible axisymmetric flows[8,20].

4.2 Application of the analogy for design of Supercritical airfoils

The outlined idea of the inclined tank walls avoids the technical difficulties with infinite or zero depth of the electrolytic tank, but also allows the electric continuation of the potential distribution beyond the sonic line in order to establish a certain distribution on it, see Fig. 12. While the tank with water as conductor is a very accurate way to model the analogy, there are other possibilities avoiding this "wet" technique and still of acceptable accuracy. One is the use of conducting graphite paper. It is, of course, of constant thickness and has therefore to be inhomogenized in order to simulate variable tank depth. Perforation of the graphite paper was used in some experiments[21] with transonic flow analog representation. It is the first step into the discretization of an electric network. This is an expensive tool if the grid is fine enough, it requires automated evaluation, being part therefore of a hybrid computational system including a digital computer. A network for solution of transonic hodograph problems was used in France[22] where the analogy has a long tradition.

The present author used the less expensive possibilities given by the graphite conducting paper. At 1970 digital computational codes for transonic potential flow analysis were just beginning to appear, design methods were not available. A project at the DFVLR in Germany was, therefore, the development of computational methods for transonic flow with the aid of the analogy[23]. This led to experiments with simple set-ups and data evaluation on the digital computer. A simple simulation of the rheoelectric tank with variable depth was achieved with acceptable accuracy through the use of compressed sheets of graphite paper, the sheets shaped parallel to isotachs in the analogy plane ζ in order to simulate $K(\upsilon)$. Fig. 13 shows a cut view through a rectangular "dry tank" outfitted with a grid of probes. Between top and bottom plate, and an elastic cushion layer, the graphite paper sheets are placed. The basic sheet extends into the regime $\upsilon > 0$ of the Rheograph plane, where the flow may be influenced by source distributions. The basic sheet also is provided with electrodes for singularity representation, in the case of a subsonic lifting airfoil with a quadrupole in order to represent a dipole with arbitrary orientation.

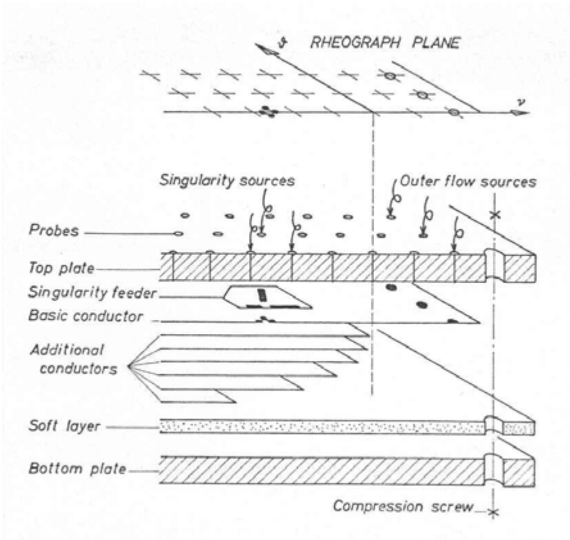
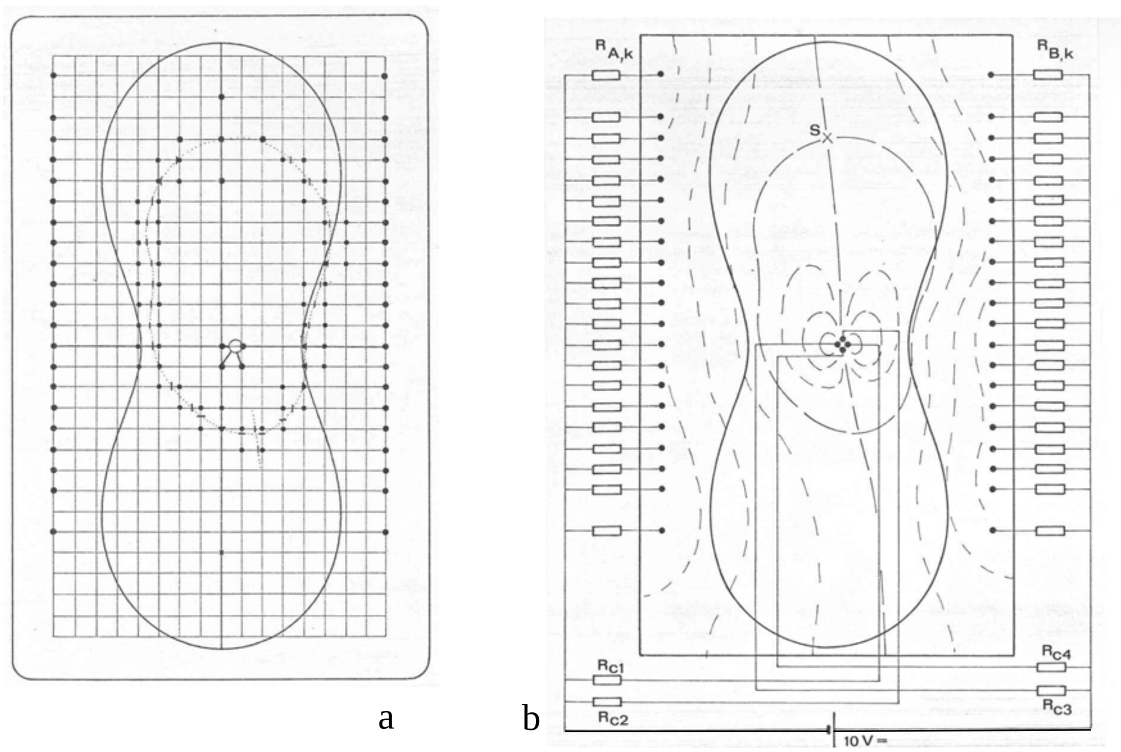


Figure 13. Analog flow table using compressed graphite paper

A set-up for Analogy B has been established and, for airfoil design application, Rheograph ζ_2 is the working plane with the useful relations (31) - (34) for lifting airfoils. In Fig. 14a the grid of probes is drawn, with external flow and singularity feeding electrodes. The latter are placed into a nearly parallel electric flow and create a line of constant $E(\equiv \Psi)$ with saddlepoints and forming a closed domain, Fig. l4b.



a b

Figure 14. Analog flow table for Rheograph ζ_2. Evaluation grid, equipotential line interpolation (a). Analog flow in ζ_2. Exterior flow and singularity, (b).

One of the saddlepoints is shifted into the mapping of the stagnation point, S, by varying the potentiometers R_A, R_B. The subsonic flow domain is shown by drawing of the sonic line lemniscate. If the enclosed domain extends outside of it, then the domain may be evaluated as an elliptic continuation analog airfoil flow representation. The line Ψ = const. is located in a limited number of intervals between probe grid points, Fig. 14 a, and the potential values are transferred to the digital computer, along with the (manually) selected position of these probes. Interpolation, potential gradient evaluation and data spline fitting along $\Psi = 0$ and along the sonic line, as well as integration of the supersonic field, boundary layer computation and viscous displacement subtraction, is carried out in the digital computer. The analog part of this hybrid technique is shown in Fig. 15.

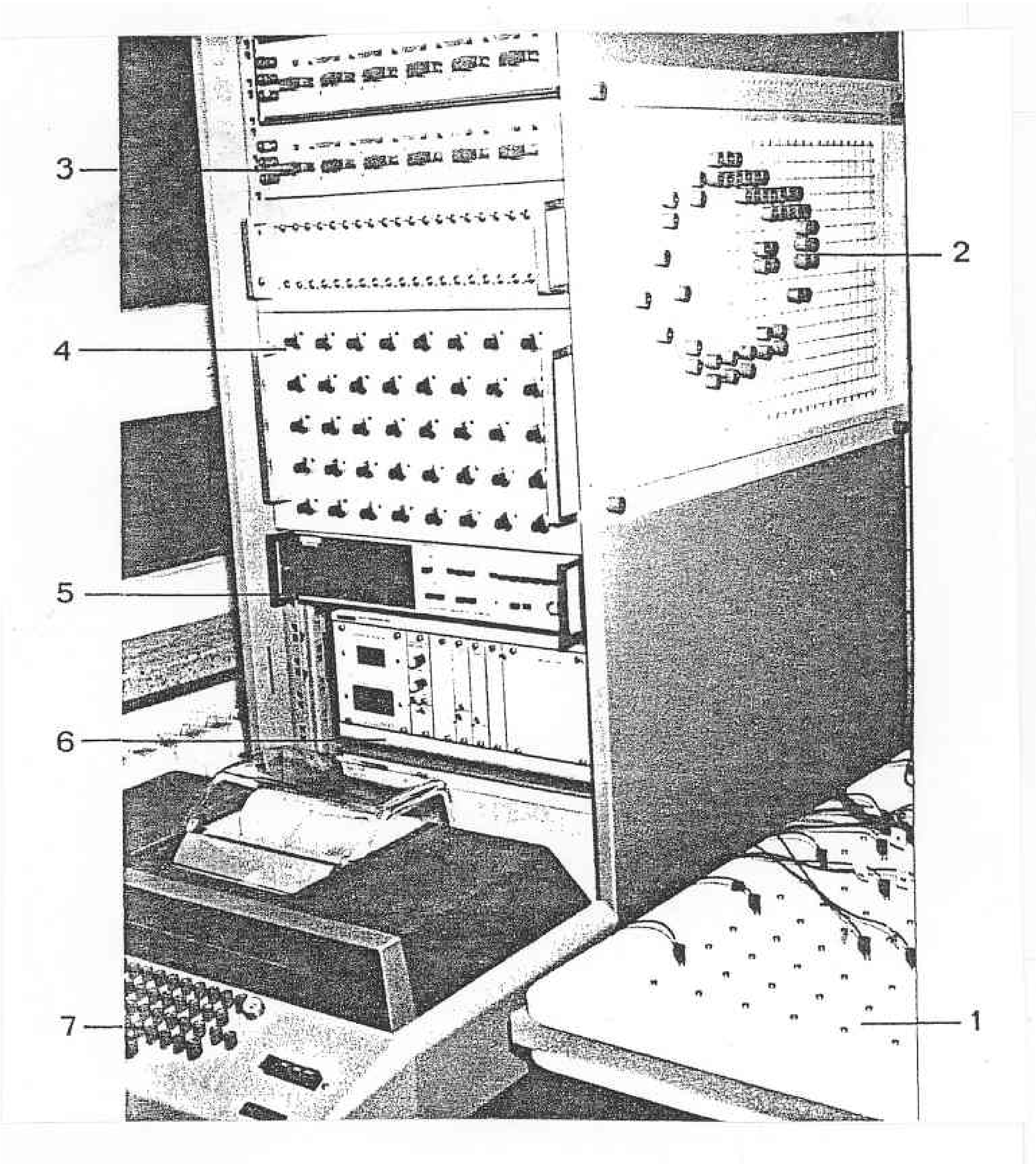


Figure 15. Analog flow evaluation with Analog flow table (1), Solution orientation (2), Bridge circuit (3), Flow adjustment Potentiometers (4), Digital Voltmeter (5), Data transfer unit and Scanner (6), Terminal and tape punch (7)

The method outlined was intended to give information about possible simplifications of hodograph techniques in the transonic regime. Availability of numerical analysis codes at a time when the first design results were obtained, accelerated the improvements and some airfoil designs were obtained for further use in supercritical wing design[24]. A result is shown in Fig. 16, the airfoil was tested in the DFVLR Göttingen Transonic Wind Tunnel[25].

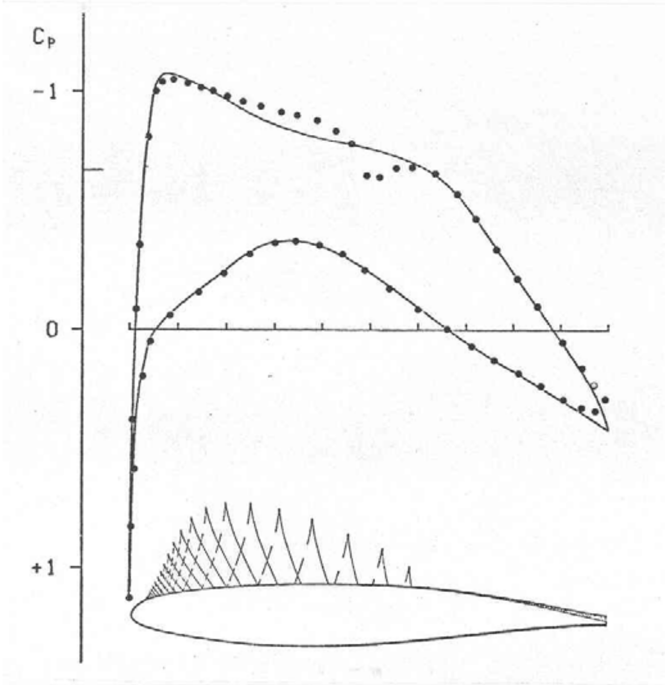


Figure 16. DFVLR 48080 Airfoil
------- Theory: M_∞ = 0.73, Re = 10Mill., C_L = 0. 53
• Experiment: M_∞= 0. 755, Re = 2.4Mill., C_L = 0. 53

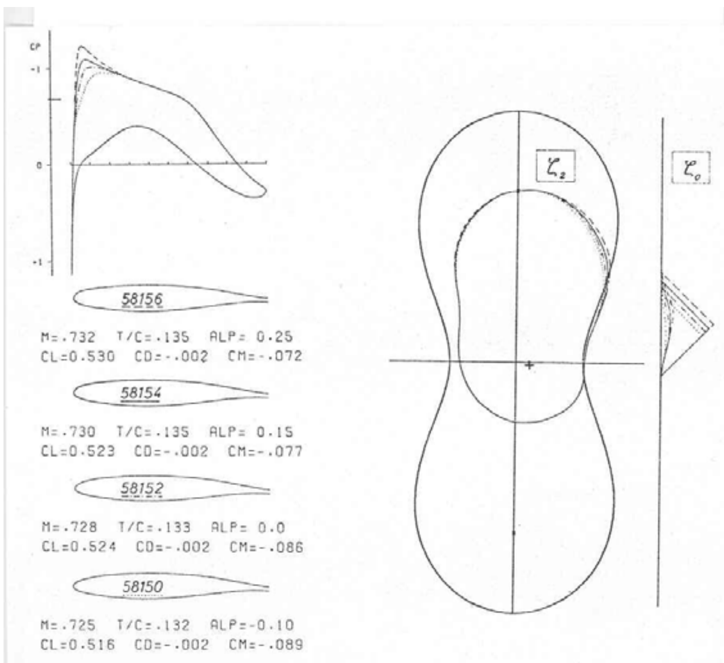
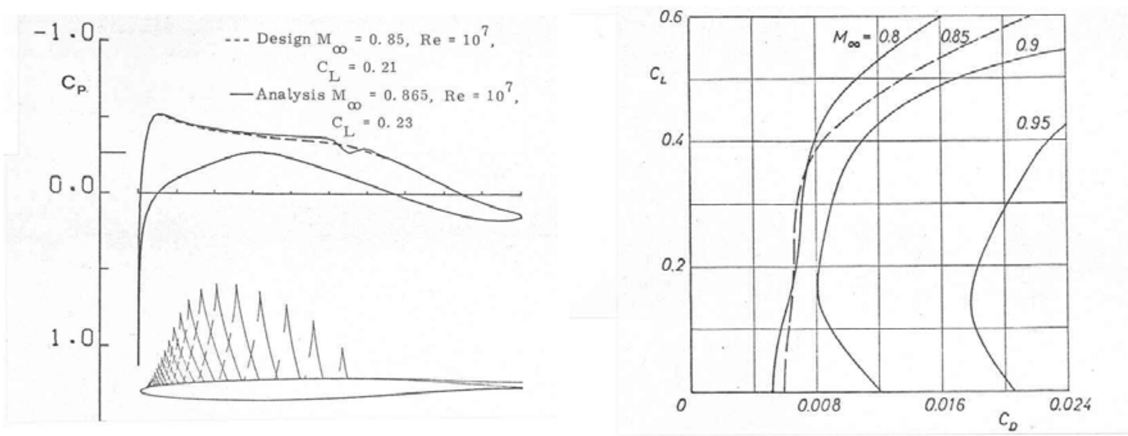


Figure 17. Variation of pressure peak

The tested airfoil is part of a series of designs differing only in the pressure peak region, Fig. 17. This was achieved by a local deformation of the analog flow airfoil mapping in the appropriate area of ζ_2. The characteristic triangles for supersonic flow field integration in ζ_0 are also drawn.

For airfoils designs with a Mach number $M_\infty > 0.82$ difficulties in the evaluation occur as a result of the slender "waist" of the sonic line lemniscate. The Rheograph plane (Fig. 7b) results from a projected $c_l = 0.2$ and $M_\infty = 0.85$. The resulting airfoil is 5.4% thick, it is drawn in Fig. 18a with the designed supersonic region. Analysis calculations were carried out for slightly different Mach numbers and angles of attack, with the result that useful designs obviously can be obtained but design Mach number and angle of attack differ somewhat from the values defining singularity location in the analog flow. The airfoil was tested in the Braunschweig Transonic Wind Tunnel[26], some of the experimental polars are shown in Fig. 18b.



a b

Figure 18. DFVLR 49201 Airfoil: Pressure distribution, supersonic domain at design conditions, (a); Drag polars for different Mach numbers, experiment: Re = 4Mill., (b).